

Question (i) Define convergent, divergent and monotonic sequence, Cauchy sequence.

Answer (i) Convergent Sequence: — If $a_n \rightarrow A$ we say that the sequence $\{a_n\}$ is convergent and that it converges to the limit A .

Thus a sequence $\{a_n\}$ is said to converge to the limit

if given $\epsilon > 0$, $\exists m \in \mathbb{N}$ such that $n \geq m \Rightarrow |a_n - A| < \epsilon$.

(ii) Divergent Sequence: — The sequence $\{a_n\}$ is said to tend to $+\infty$, if corresponding to a positive number M , however large, a positive number m can be found such that

$$a_n < -M \text{ when } n \geq m.$$

We write $a_n \rightarrow +\infty$ or $-\infty$ as $n \rightarrow \infty$

$$\text{or } \lim_{n \rightarrow \infty} a_n = +\infty \text{ or } -\infty$$

In both the cases $\{a_n\}$ is a divergent sequence and we say that a_n diverges to $+\infty$ or $-\infty$.

Thus if $a_n \rightarrow +\infty$ or $-\infty$, the sequence is said to be divergent.

(iii) Monotonic Sequence: Suppose the sequence $\{a_n\}$ be such that

$$\text{either } a_1 \leq a_2 \leq a_3 \leq \dots \leq a_n \leq a_{n+1} \dots$$

$$\text{or } a_1 \geq a_2 \geq a_3 \geq \dots \geq a_n \geq a_{n+1} \dots$$

then it is said to be monotonic

In the first case, each element is equal to or greater than the preceding element i.e., $a_n \leq a_{n+1}$ for all n , the

sequence is said to be monotonic increasing and in the second case, each element is equal to or less than the preceding element i.e., $a_n \geq a_{n+1}$ i.e., $a_n \geq a_{n+1} \forall n$, the sequence is said to be monotonic decreasing.

(iv) CAUCHY SEQUENCE: defn: — A sequence $\{a_n\}$ of real numbers is

called a Cauchy sequence, if given $\epsilon > 0$, there exists a natural number p such that

$$m, n \geq p \Rightarrow |a_n - a_m| < \epsilon.$$

In other words, a sequence is a Cauchy sequence if the terms of the sequence become arbitrarily close to each other after a certain stage.